Discrete Symmetries and Anomalies in String Models

Tatsuo Kobayashi

1. Introduction
2. Abelian Discrete Symmetries
3. Non-Abelian Discrete Symmetries
4. Anomalies
5. Summary

based on collaborations with

H.Abe, T.Araki, K.S.Choi, P.Ko, J.Kubo, H.P.Nilles, H.Ohki, J.H.Park, F.Ploger, S.Raby, S.Ramos-Sanches, M.Ratz, M.Sakai, P.Vaudrevange, R.J.Zhang

1. Introduction

Now, we have lots of 4D string models leading to
(semi-)realistic massless spectra such as

SU(3)xSU(2)xU(1) gauge groups,
three chiral genenations,
vector-like matter fields and lots of singlets
with and without chiral exotic fields,

e.g. in
heterotic orbifold models,
type II intersecting D-brane models,
type II magnetized D-brane models,
etc.

What about their 4D low-energy effective theories ?
Are they realistic ?
What about the quark/lepton masses and mixing angles ?

4D low-energy effective field theory

Abelian discrete symmetries

In general, string models lead to Abelian discrete symmetries, which are quite important to control 4D low-energy effective field theory.

Quark/Lepton masses and mixing angles

The top quark mass, i.e. O(1) of Yukawa coupling, can be derived in many string models.

How to derive other light fermion masses (corresponding to suppressed Yukawa couplings) is model-dependent.

Flavor physics is still a challenging issue.

Lepton masses and mixing angles

$$M_e = 0.5 \quad MeV, \qquad M_\mu = 106 \quad MeV$$

$$M_\tau = 1.8 \quad GeV,$$

mass squared differences and mixing angles consistent with neutrino oscillation

$$\Delta M_{21}^2 = 8\times 10^{-5} \quad eV^2, \qquad \Delta M_{31}^2 = 2\times 10^{-3} \quad eV^2$$

$$\sin^2\theta_{12} = 0.3, \qquad \sin^2\theta_{23} = 0.5, \qquad \sin^2\theta_{13} = 0,$$

large mixing angles

Tri-bimaximal mixing Ansatz

$$V_{MNS} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

large mixing angles

Non-Abelian discrete flavor symm.

Recently, in field-theoretical model building, several types of discrete flavor symmetries have been proposed with showing interesting results,

e.g. S3, D4, A4, S4, Q6, Δ(27), ...

Review: e.g

Ishimori, T.K., Ohki, Okada, Shimizu, Tanimoto '10

⇒ large mixing angles

one Ansatz: tri-bimaximal

$$\begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \end{pmatrix}$$

Non-Abelian symm.

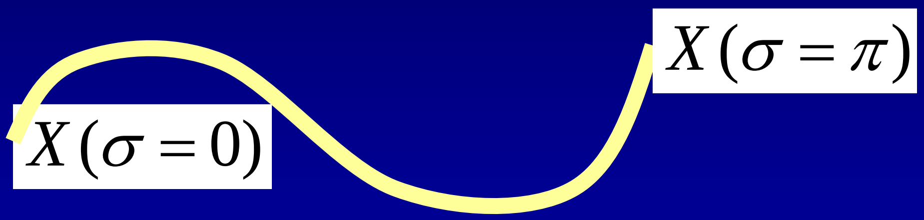
String model builders have not cared about non-Abelian discrete symmetires.

Recently, we showed that certain non-Abelian flavor symmetries appear in string models.

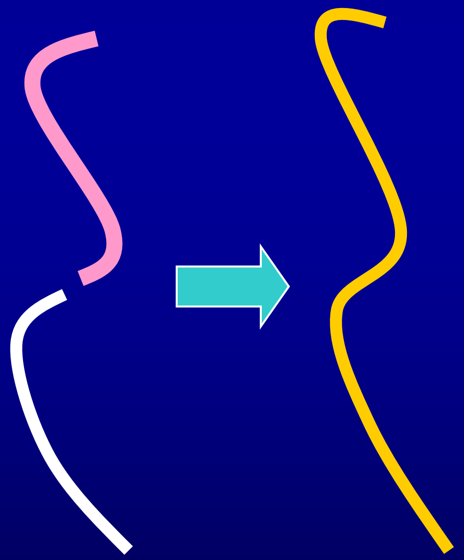
Studies on discrete anomalies are also important.

2. Abelian discrete symmetries

2-1. coupling selection rule



A string can be specified by its boundary condition.



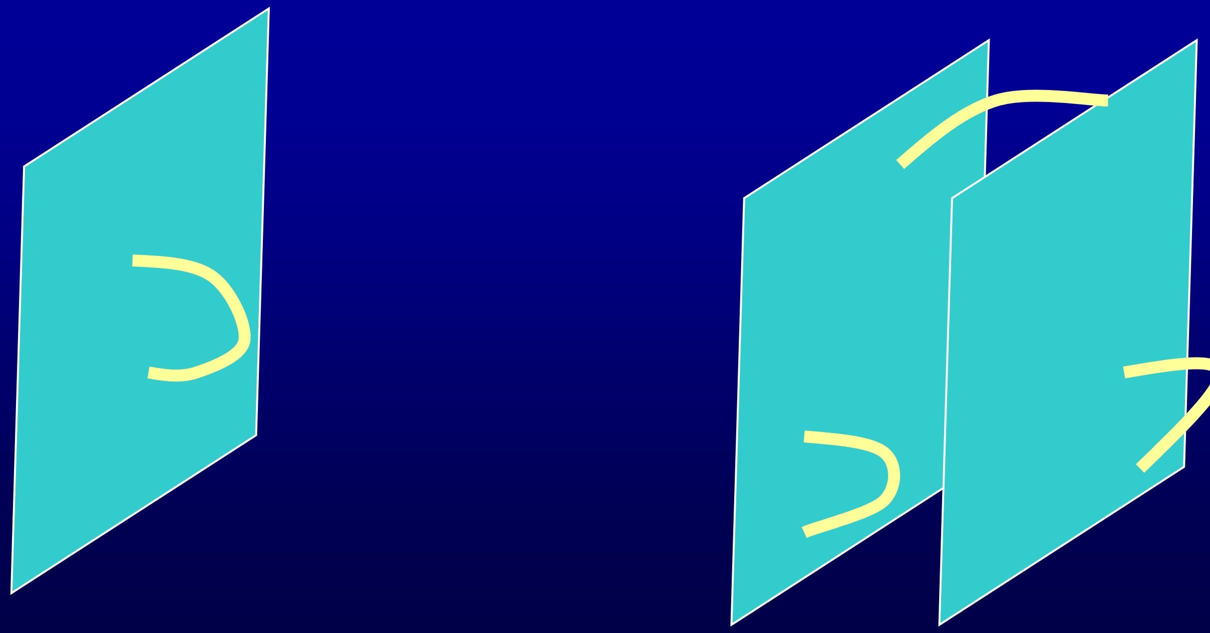
Two strings can be connected to become a string if their boundary conditions fit each other.

→ coupling selection rule symmetry

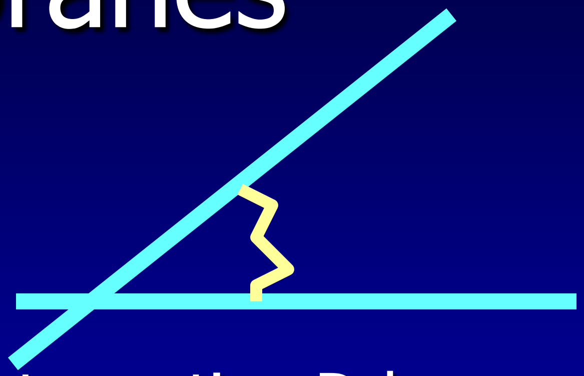
2-2. Intersecting D-brane models

gauge boson: open string, whose two end-points are on the same (set of) D-brane(s)

N parallel D-branes ⇒ U(N) gauge group



Intersecting D-branes



Where is matter fields ?

New modes appear between intersecting D-branes.

They have charges under both gauge groups, i.e. bi-fundamental matter fields.

boundary condition

$$X^2(\sigma = 0) = 0, \quad \partial_\sigma X^1(\sigma = 0) = 0$$

$$X^1(\sigma = \pi)\tan\theta\pi + X^2(\sigma = \pi) = 0,$$

$$\partial_\sigma X^1(\sigma = \pi) - \partial_\sigma X^2(\sigma = \pi)\tan\theta\pi = 0$$

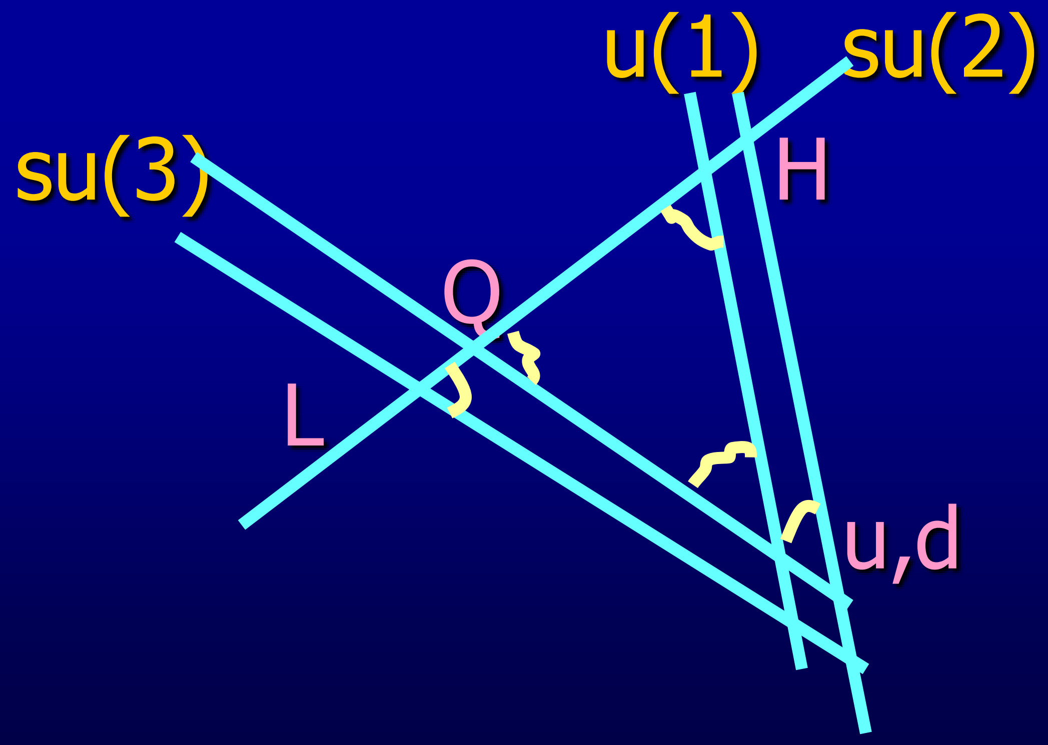
Twisted boundary condition

Toy model (in uncompact space)

gauge bosons : on brane

quarks, leptons, higgs :

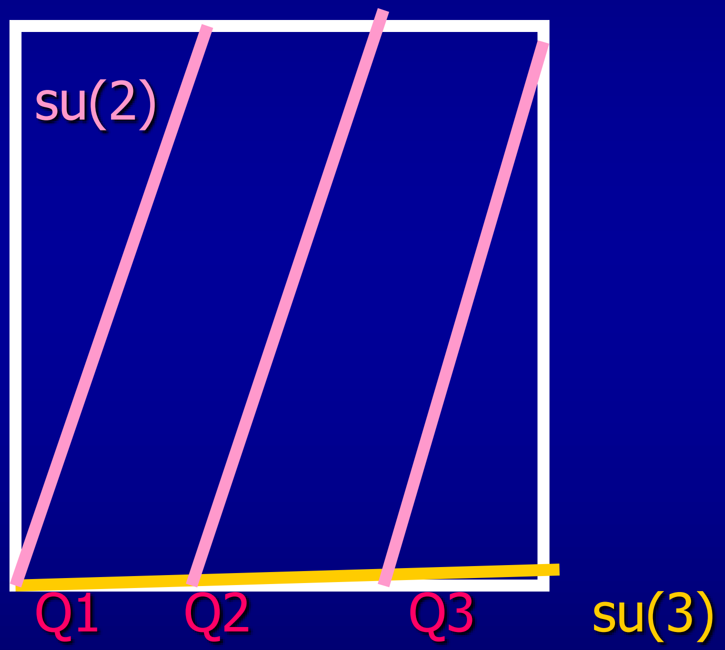
localized at intersecting points

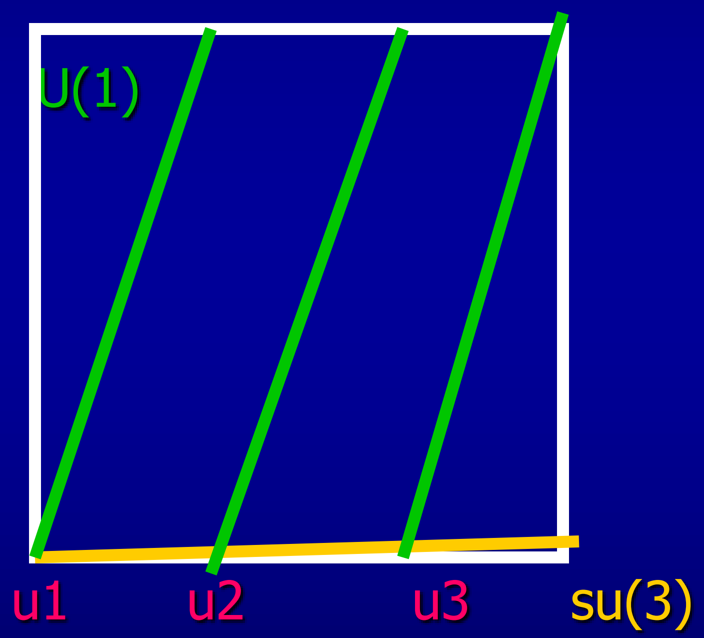


Generation number

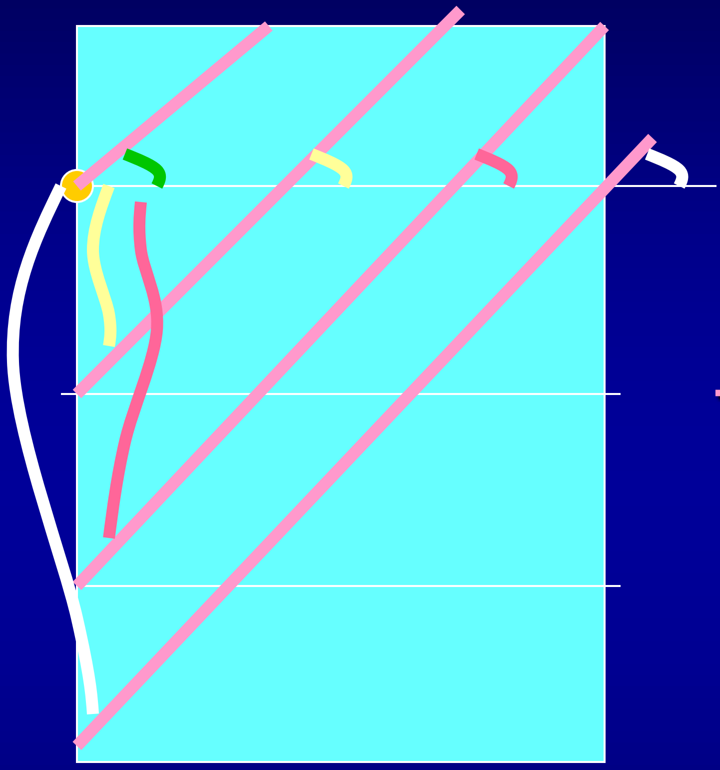
Torus compactification

Family number = intersection number





Boundary conditions



$$X^2(\sigma=0)-X^2(\sigma=\pi)$$
$$=0,1,2 \pmod 3$$

Three strings with the same gauge charges can be distinguished by boundary conditions, i.e. Z_3 charges.

Generic case ⟹ Z_N symmetries

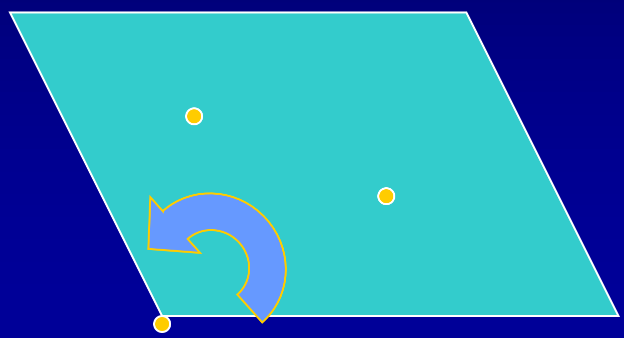
2.3 Heterotic orbifold models

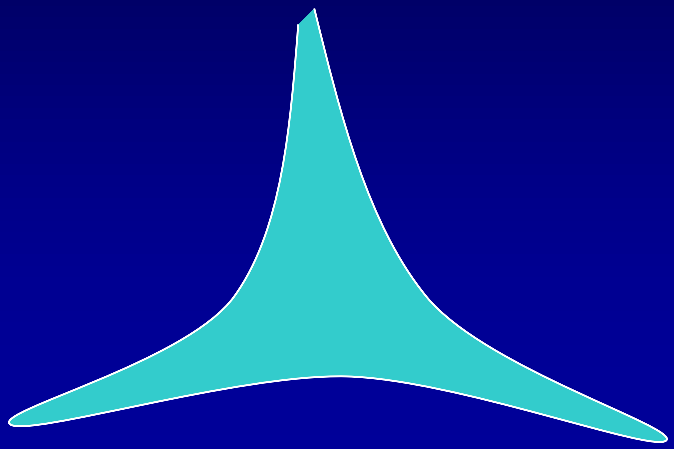
S1/Z2 Orbifold

There are two singular points,
which are called fixed points.

Orbifolds

T2/Z3 Orbifold





There are three fixed points on Z3 orbifold

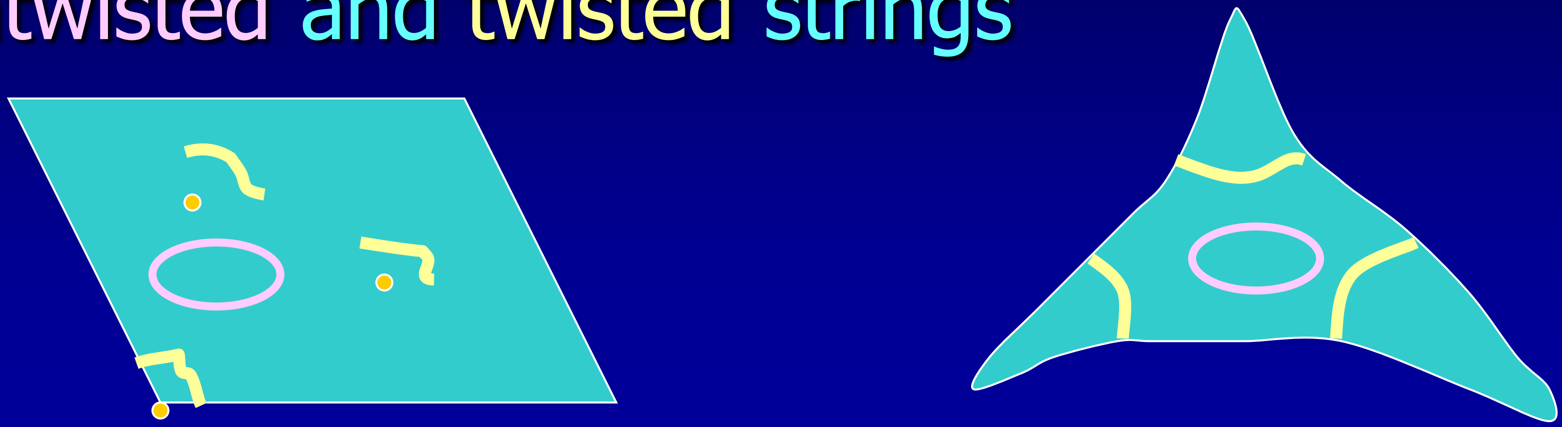
(0,0), (2/3,1/3), (1/3,2/3) su(3) root lattice

Orbifold = D-dim. Torus /twist

Torus = D-dim flat space/ lattice

Closed strings on orbifold

Untwisted and twisted strings



Twisted strings are associated with fixed points.

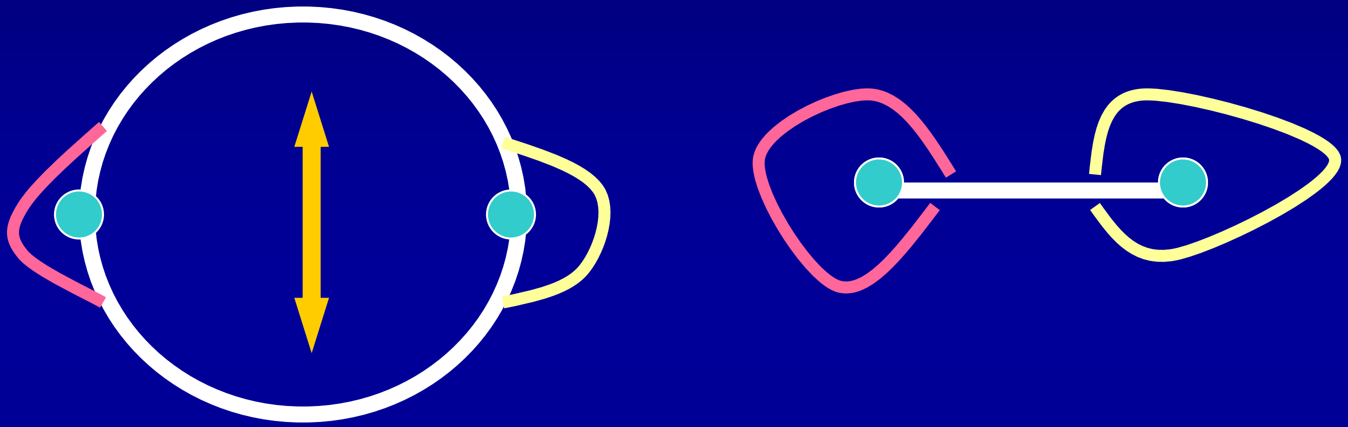
"Brane-world" terminology:

untwisted sector   bulk modes

twisted sector   brane (localized) modes

Heterotic orbifold models

S1/Z2 Orbifold



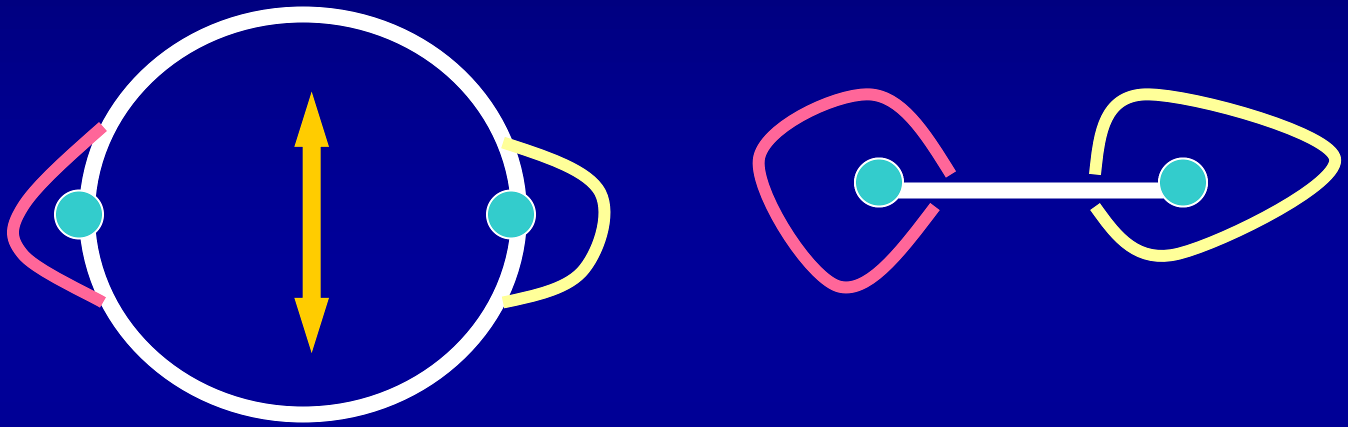
$$X(\sigma=\pi) = -X(\sigma=0)$$

$$X(\sigma=\pi) - e/2 = -(X(\sigma=0) - e/2)$$

$$X(\sigma=\pi) = -X(\sigma=0) + n\,e, \quad n = 0,1 \ (\mathrm{mod}\ 2)$$

Heterotic orbifold models

S1/Z2 Orbifold



twisted string

$$X(\sigma=\pi)=-X(\sigma=0)+n\,e,\quad n=0,1\ (\mathrm{mod}\ 2)$$

untwisted string $\quad X(\sigma=\pi)=X(\sigma=0)$

$$X(\sigma=\pi)=(-1)^m X(\sigma=0)+n\,e,$$

$$m,n=0,1\ (\mathrm{mod}\ 2)$$

Z2 x Z2 in Heterotic orbifold models

S1/Z2 Orbifold

$$X(\sigma=\pi) = (-1)^m X(\sigma=0) + n\, e,$$

$$m, n = 0, 1 \pmod 2$$

two Z2's

twisted string

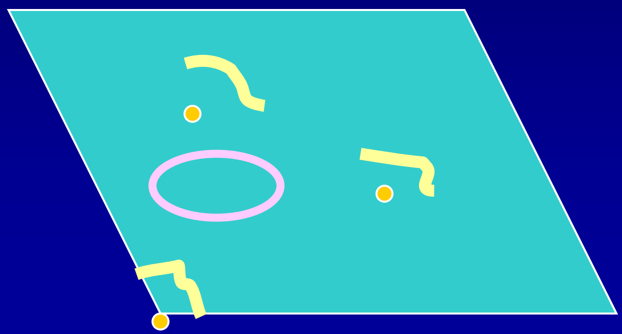
$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

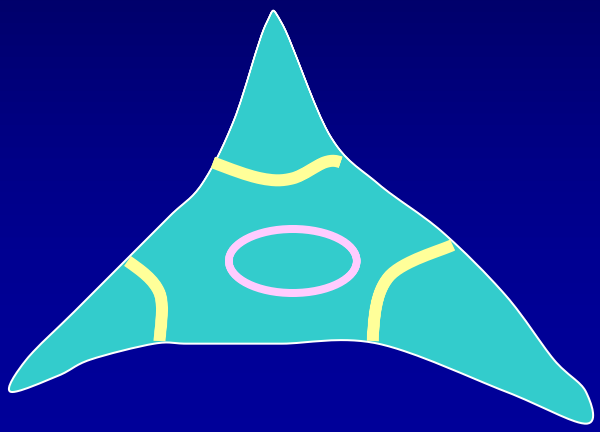
untwisted string

Z2 even for both Z2

Closed strings on orbifold

Untwisted and twisted strings





Twisted strings (first twisted sector)

$$X(\sigma=\pi)=\theta X(\sigma=0)+n\,e_1,\quad n=0,1,2\,(\mathrm{mod}\ 3)$$

$$\theta=120^\circ \text{ twist,}\quad \text{up to lattice } \Lambda=3me_1+n(e_1-e_2)$$

second twisted sector

$$X(\sigma=\pi)=\theta^2 X(\sigma=0)+n\,e_1,\quad n=0,1,2\,(\mathrm{mod}\ 3)$$

untwisted sector

$$X(\sigma=\pi)=X(\sigma=0)$$

Z3 x Z3 in Heterotic orbifold models

T2/Z3 Orbifold

$$X(\sigma=\pi)=\theta^m X(\sigma=0)+n\,e,$$

$$m,n=0,1,2 \qquad (\mathrm{mod}\ 3)$$

two Z3's

twisted string (first twisted sector)

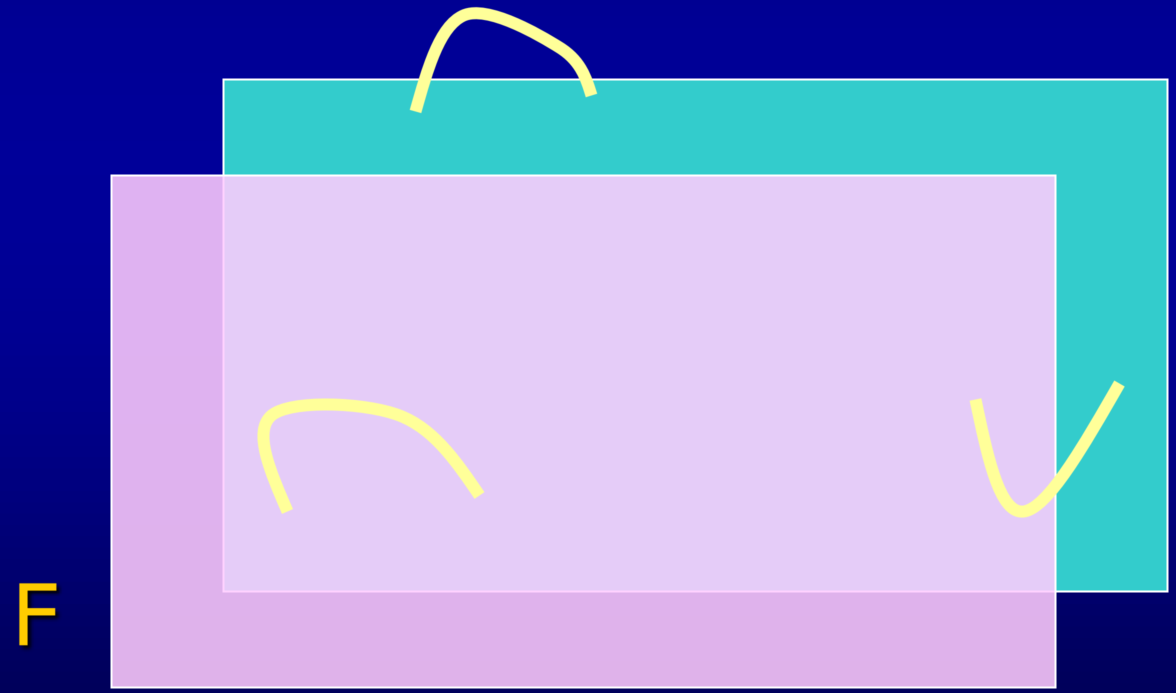
$$\begin{pmatrix}\omega & 0 & 0\\ 0 & \omega & 0\\ 0 & 0 & \omega\end{pmatrix},\quad \begin{pmatrix}1 & 0 & 0\\ 0 & \omega & 0\\ 0 & 0 & \omega^2\end{pmatrix},\quad \omega=\exp(2\pi i/3)$$

untwisted string

vanishing Z3 charges for both Z3

2-4. Magnetized D-branes

We consider torus compactification
with magnetic flux background.



Boundary conditions on magnetized D-branes

$$\partial_\sigma X^4 + F_{45}\partial_\tau X^5 = 0,$$
$$F_{45}\partial_\tau X^4 - \partial_\sigma X^5 = 0,$$

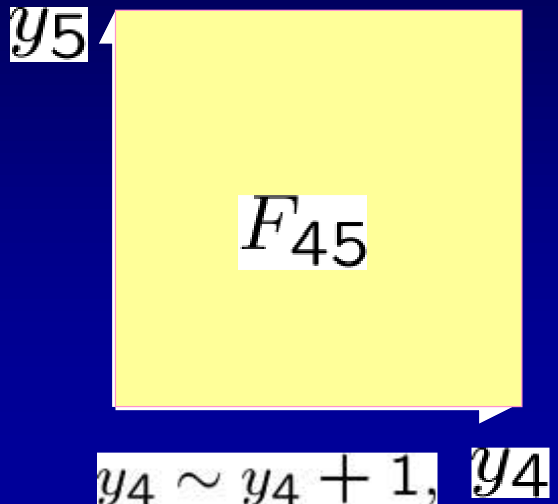
similar to the boundary condition of open string between intersecting D-branes

T-dual

Higher Dimensional theory with flux

$U(1)$

Abelian gauge field on magnetized torus T^2



$y_4 \sim y_4 + 1,$
$y_5 \sim y_5 + 1$

Constant magnetic flux $F_{45} = b,$

gauge fields of background $\begin{cases} A_4 = 0, \\ A_5 = b y_4 \end{cases}$

⇓

Consistency requires Dirac's quantization condition.

$$\frac{b}{2\pi} = M \in Z$$

Torus with magnetic flux

We solve the zero-mode Dirac equation,

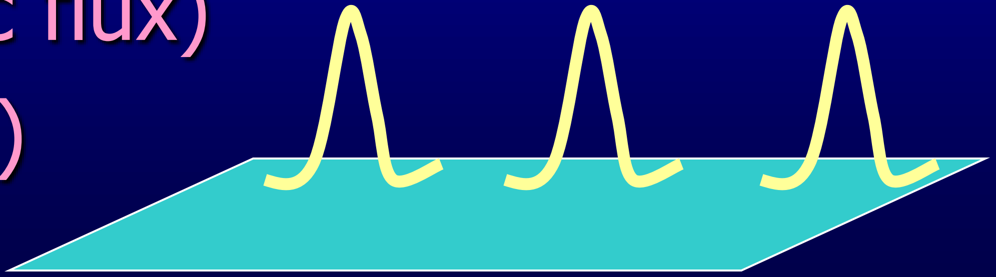
$$i\gamma^m D_m \psi = 0$$

e.g. for U(1) charge q=1.

Torus background with magnetic flux leads to chiral spectra.

the number of zero-modes

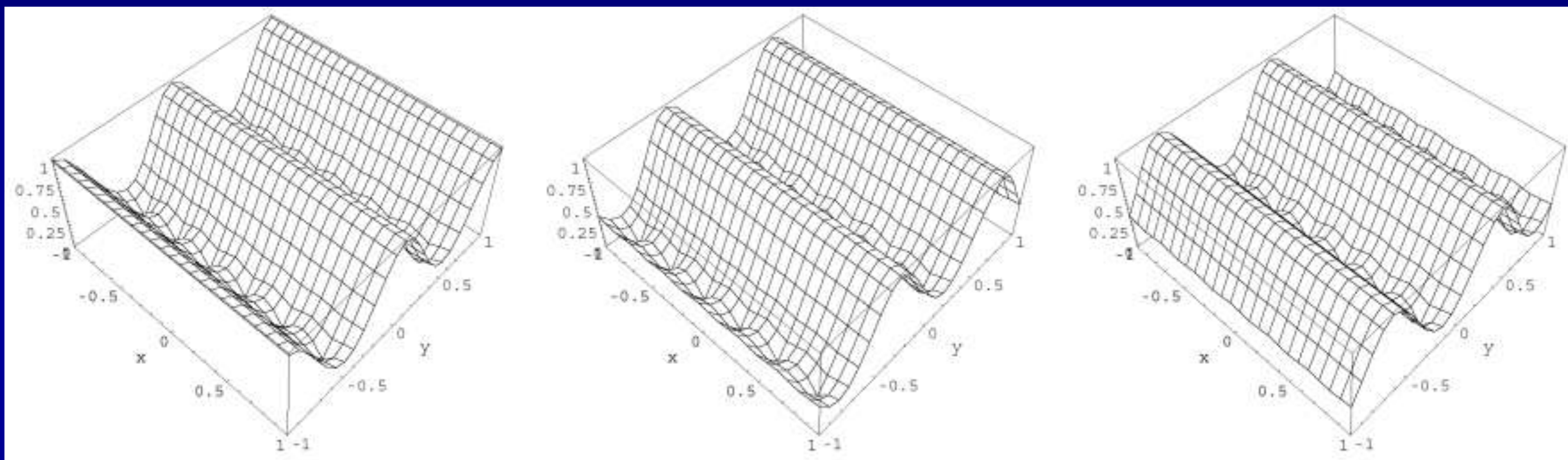
= M (magnetic flux) x q (charge)



Wave functions

For the case of M=3

$|\Theta^0(y)|$ $|\Theta^1(y)|$ $|\Theta^2(y)|$



Wave function profile on toroidal background

Zero-modes wave functions are quasi-localized far away each other in extra dimensions. Therefore the hierarchirally small Yukawa couplings may be obtained.

Zero-modes

$$F_{45} = 2\pi M, \quad A_4 = 0, \quad A_5 = 2\pi y_4$$

Wave-function = (gaussian) x (theta-function)

We have quantized momentum,

$$P_5 = 2\pi k, \quad (\mathrm{mod} \;\; \mathrm{M})$$

The peaks of wave functions correspond to

$$y_4 = k/\mathrm{M}$$

The momentum conservation

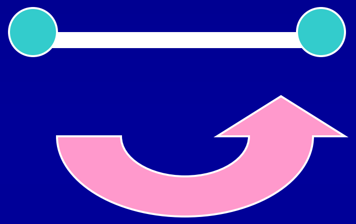
→ ZM discrete symmetry

e.g. M=3 → Z3 symmetry

3. Non-Abelian discrete symmetries

3-1. Heterotic orbifold models

S1/Z2 Orbifold



String theory has two Z2's.

In addition, the Z2 orbifold has the geometrical symmetry, i.e. Z2 permutation.

$$X(\sigma = \pi) = (-1)^m X(\sigma = 0) + n\, e,$$

$$m, n = 0, 1 \pmod 2$$

D4 Flavor Symmetry

Stringy symmetries require that Lagrangian has the permutation symmetry between 1 and 2, and each coupling is controlled by two Z2 symmetries.

Flavor symmeties: closed algebra S2 U(Z2xZ2)

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad -1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

D4 elements

$$\pm 1, \qquad \pm \sigma_1, \qquad \pm i\sigma_2, \qquad \pm \sigma_3$$

modes on two fixed points ⇒ doublet

untwisted (bulk) modes ⇒ singlet

Geometry of compact space

→ origin of finite flavor symmetry

Abelian part (Z2xZ2) : coupling selection rule

S2 permutation : one coupling is the same as another.

T.K., Raby, Zhang, '05

Explicit Z6-II model: Pati-Salam

T.K. Raby, Zhang '04

Z6-II includes 2D Z2 orbifold.

Once we fix the orbifold and gauge background in string theory, all of modes can be computed. One can not add or reduce any modes by hand (unlike field-theoretical brane-world models).

Gauge group

$$SU(4)\times SU(2)\times SU(2)\times SO(10)'\times SU(2)'\times U(1)^5$$

Chiral fields

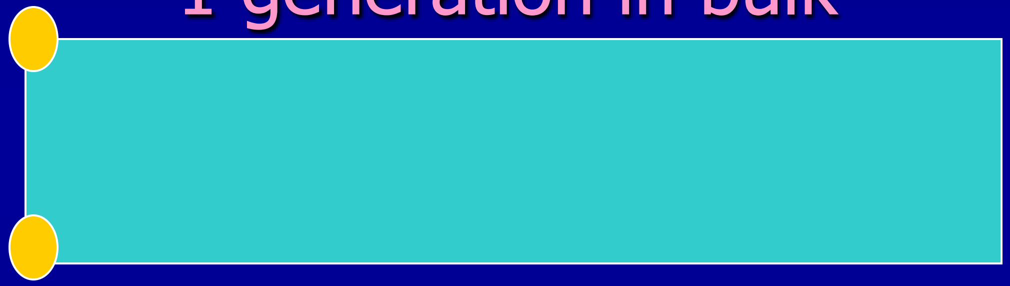
Pati-Salam model with 3 generations + extra fields

All of extra matter fields can become massive

Heterotic orbifold as brane world

2D Z2 orbifold

1 generation in bulk



two generations on two fixed points

unbroken $SU(4) * SU(2) * SU(2)$ D4

bulk $\Rightarrow$ (4,2,1) + (4*,1,2)+... singlet

localized modes $\Rightarrow$ (4,2,1) + (4*,1,2) doublet

Explicit Z6-II model: MSSM

Buchmuller, Hamaguchi, Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, '06, '07

4D massless spectrum

Gauge group $SU(3) \times SU(2) \times U(1)_Y \times G_H$

Chiral fields

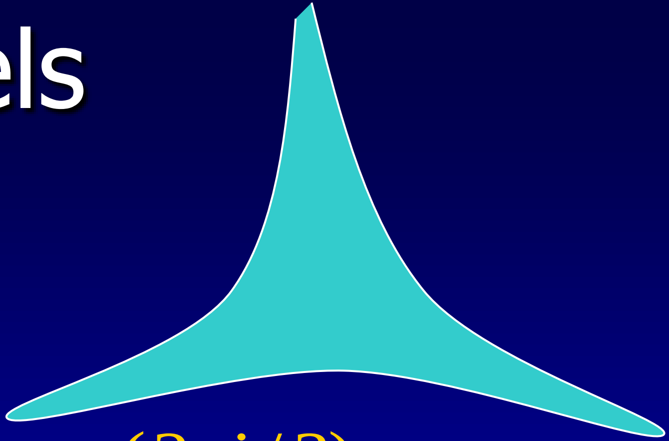
3 generations of MSSM + extra fields

All of extra matter fields can become massive along flat directions

There are O(100) models.

Heterotic orbifold models

T2/Z3 Orbifold



two Z3's

$$\begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad \omega = \exp(2\pi i/3)$$

Z3 orbifold has the S3 geometrical symmetry,

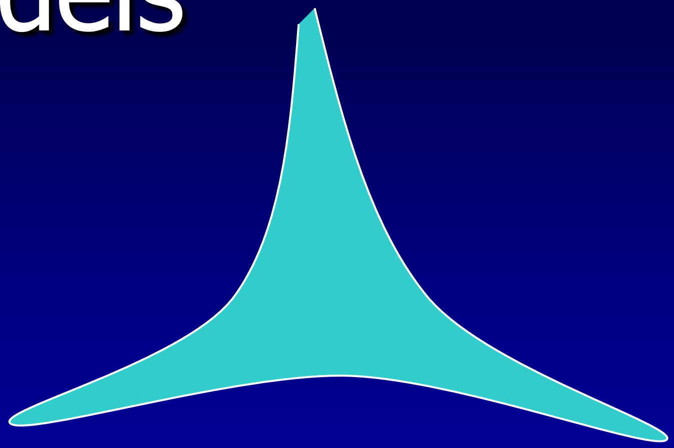
$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Their closed algebra is Δ(54).

T.K., Nilles, Ploger, Raby, Ratz, '07

Heterotic orbifold models

T2/Z3 Orbifold



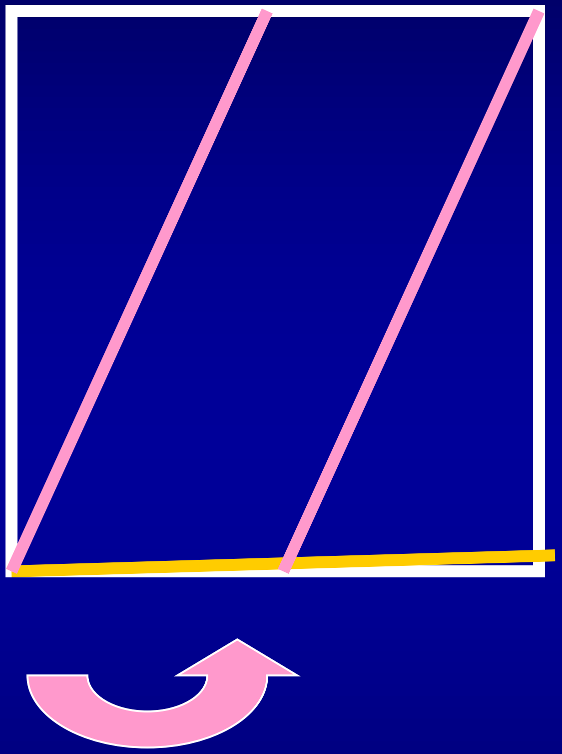
has Δ(54) symmetry.

localized modes on three fixed points → Δ(54) triplet

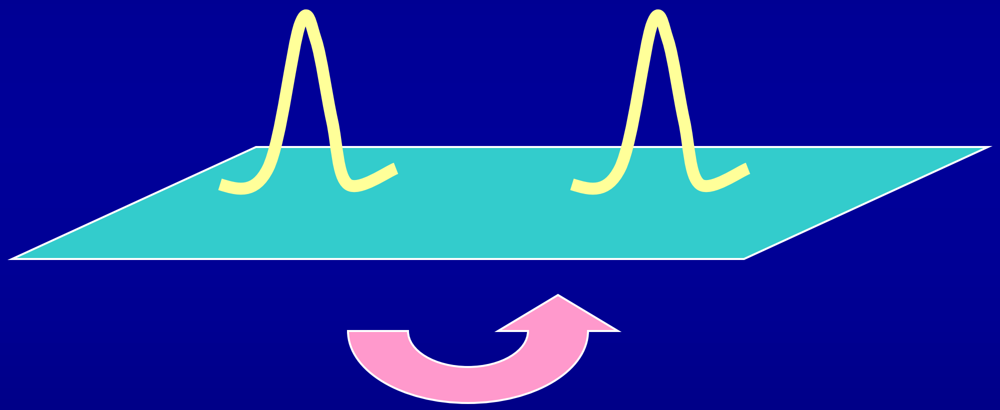
bulk modes → Δ(54) singlet

T.K., Nilles, Ploger, Raby, Ratz, '07

3-2. intersecting/magnetized D-brane models



Abe, Choi, T.K. Ohki, '09, '10

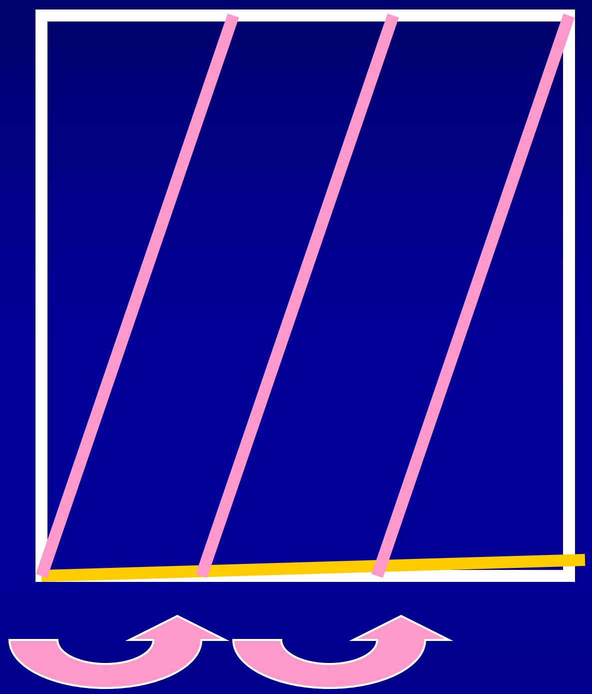


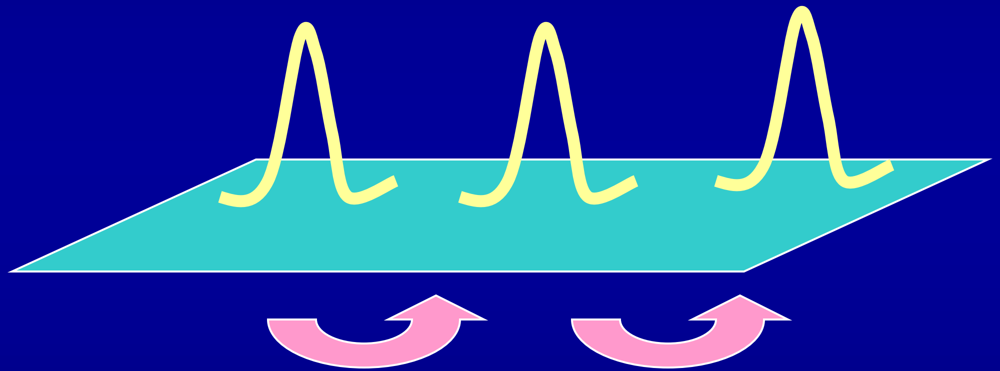
There is a Z2 permutation symmetry.

The full symmetry is D4.

intersecting/magnetized D-brane models

Abe, Choi, T.K. Ohki, '09, '10





geometrical symm. Full symm.

Z3 Δ(27)

S3 Δ(54)

intersecting/magnetized D-brane models

Abe, Choi, T.K. Ohki, '09, '10

generic intersecting number g

magnetic flux

flavor symmetry is a closed algebra of

two Zg's.
$$\begin{pmatrix} 1 & & & \\ & \rho & & \\ & & \ddots & \\ & & & \rho^{g-1} \end{pmatrix}, \quad \begin{pmatrix} \rho & & & \\ & \rho & & \\ & & \ddots & \\ & & & \rho \end{pmatrix}, \quad \rho = e^{2\pi i/g}$$

and Zg permutation
$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & & & \\ & & & \end{pmatrix}$$

Certain case: Zg permutation larger symm. Like Dg

Magnetized brane-models

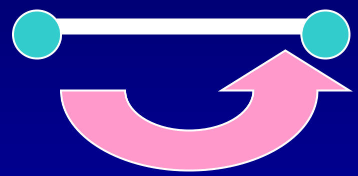
Magnetic flux M	D4
2	2
4	$1_{++} + 1_{+-} + 1_{-+} + 1_{--}$
...	....

Magnetic flux M	Δ(27) (Δ(54))
3	3_1
6	$2 \times \bar{3}_1$
9	$\sum 1_n$ $n=1,\ldots,9$
	$(1_1+\sum 2_n$ $n=1,\ldots,4)$
...	....

3-3. field theory: extension

Abe, Choi, T.K., Ohki, Sakai, '10

S1/Z2 Orbifold



geometrical symm.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

String theory has two Z2's.

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

We assign generic ZN charges to localized fields on two fixed points,

$$\begin{pmatrix} e^{2\pi iq/N} & 0 \\ 0 & e^{2\pi ip/N} \end{pmatrix}$$

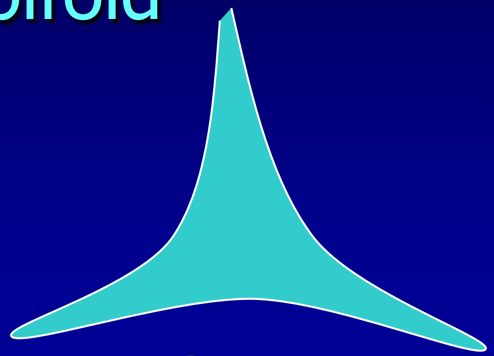
→ flavor symmetries $S_3,\ D_N,\ \Sigma(2N^2)$

field theory: extension

T2/Z3 Orbifold

geometrical symm.

Z3, S3



String theory has two Z3's.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

We assign generic ZN charges to localized fields on three fixed points,

$$\begin{pmatrix} e^{2\pi i p/N} & 0 & 0 \\ 0 & e^{2\pi i q/N} & 0 \\ 0 & 0 & e^{2\pi i r/N} \end{pmatrix}$$

→ flavor symmetries

$$A_4,\ \Delta(3N^2),\ S_4,\ \Delta(6N^2), Q_N,\ T_7,\ \Sigma(81), \cdots$$

Stringy derivation is not clear.

4. Discrete anomalies

4-1. Abelian discrete anomalies

Symmetry → violated

quantum effects

U(1)-G-G anomalies

anomaly free condition

$$\sum q\, T_2(R) = 0$$

ZN-G-G anomalies

anomaly free condition

$$\sum q\, T_2(R) = 0 \qquad (\mathrm{mod}\ \ N)$$

Abelian discrete anomalies: path integral

Zn transformation $\psi \to \psi'$

path integral measure

$$D\psi D\overline{\psi} \to J\ D\psi D\overline{\psi}$$

$$J = \exp[A \tfrac{i}{32\pi^2}\int d^4x\ \mathrm{tr}(F^{\mu\nu}\tilde{F}_{\mu\nu})]$$

$$A = \frac{1}{N}\sum q\,T_2(R)$$

$$\tfrac{1}{32\pi^2}\int d^4x\ \mathrm{tr}(F^{\mu\nu}\tilde{F}_{\mu\nu}) = \text{integer}$$

ZN-G-G anomalies

anomaly free condition

$$\sum q\,T_2(R) = 0 \quad (\mathrm{mod}\ \ N)$$

Heterotic orbifold models

There are two types of Abelian discrete symmetries.

T2/Z3 Orbifold

$$X(\sigma = \pi) = \theta^m X(\sigma = 0) + n\, e,$$

$$m, n = 0, 1, 2 \qquad (\mathrm{mod}\ 3)$$

two Z3's

One is originated from twists,

the other is originated from shifts.

Both types of discrete anomalies $\sum q\, T_2(R)$ are universal for different groups G.

Araki, T.K., Kubo, Ramos-Sanches, Ratz,Vaudrevange, '08

Heterotic orbifold models

U(1)-G-G anomalies $\sum q\, T_2(R)$

are universal for different groups G.

→ 4D Green-Schwarz mechanism due to a single axion (dilaton), which couples universally with gauge sectors.

ZN-G-G anomalies may also be cancelled by 4D GS mechanism.

There is a certain relations between U(1)-G-G and ZN-G-G anomalies, anomalous U(1) generator is a linear combination of anomalous ZN generators.

Araki, T.K., Kubo, Ramos-Sanches, Ratz,Vaudrevange, '08

4-2. Non-Abelian discrete anomalies

Araki, T.K., Kubo, Ramos-Sanches, Ratz, Vaudrevange, '08

Non-Abelian discrete group

$G = \{g_1, g_2, \cdots, g_M\}$ finite elements

Each element generates an Abelian symmetry.

$$(g_k)^{N_k} = 1$$

We check ZN-G-G anomalies for each element.

$$\sum q_k T_2(R) = 0 \quad (\mathrm{mod}\ N_k)$$

All elements are free from ZN-G-G anomalies.

→ The full symmetry G is anomaly-free.

Some ZN symmetries for elements g_k are anomalous.

→ The remaining symmetry corresponds to the closed algebra without such elements.

Non-Abelian discrete anomalies

matter fields = multiplets under non-Abelian discrete symmetry

Each element is represented by a matrix on the multiplet.

$\det(g_k) = 1$ → $\sum q_k \, T_2(R) = 0 \pmod{N_k}$

Such a multiplet does not contribute to ZN-G-G anomalies.

String models lead to certain combinations of multiplets.

→ limited pattern of non-Abelian discrete anomalies

Heterotic string on Z2 orbifold: D4 Flavor Symmetry

Flavor symmeties: closed algebra S2 U(Z2xZ2)

modes on two fixed points ⇒ doublet

untwisted (bulk) modes ⇒ singlet

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad -1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The first Z2 is always anomaly-free, while the others can be anomalous.

However, it is simple to arrange models such that the full D4 remains.

e.g. left-handed and right-handed quarks/leptons

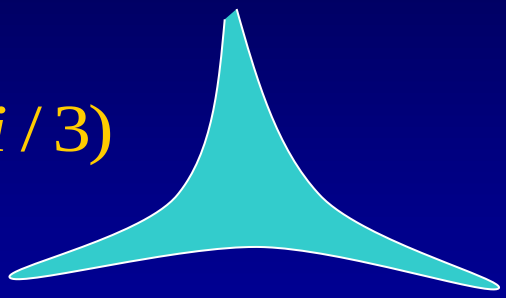
1 + 2

Such a pattern is realized in explicit models.

Heterotic models on Z3 orbifold

two Z3's

$$\begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad \omega = \exp(2\pi i/3)$$



Z3 orbifold has the S3 geometrical symmetry,

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Their closed algebra is Δ(54).

The full symmetry except Z2 is always anomaly-free.

That is, the Δ(27) is always anomaly-free.

Abe, et. al. work in progress

Magnetized/intersecting brane-models

In general, several representations appear, e.g.

Magnetic flux M	D4
2	2
4	$1_{++} + 1_{+-} + 1_{-+} + 1_{--}$
. . .	. . . .

Similar to heterotic orbifold models, only Z2 symmetries can be anomalous, but ZN symmetries with N=odd are always anomaly-free.

Abe, et. al. work in progress

4-2. Implication

Under the full symmetry, the three generations have different quantum numbers.

Kahler potential is diagonal,

$$K = K_{11}(X)|q_1|^2 + K_{22}(X)|q_2|^2 + K_{33}(X)|q_3|^2 + \cdots$$

where X denote singlet fields (moduli)

triplet $K_{11}(X) = K_{22}(X) = K_{33}(X)$

1+2 $K_{11}(X) = K_{22}(X) \neq K_{33}(X)$

1+ 1'+1" $K_{ii}(X)$: independen of each other

sfermion mass

SUSY breaking due to F-term of X

triplet

$$\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_1^2 & 0 \\ 0 & 0 & m_1^2 \end{pmatrix}$$

1+2

$$\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_1^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}$$

1+ 1'+1"

$$\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}$$

symmetry breaking

Breaking of the flavor symmetries would induce off-diagonal elements in the Kahler potential, e.g.

$$\Delta K = K_{12}(X)(q_1 q_2^* + q_1^* q_2) + \cdots$$

and sfermion mass-squared matrix,

e.g.

$$\begin{pmatrix} m_1^2 & \Delta m_{12}^2 & 0 \\ \Delta m_{21}^2 & m_1^2 & 0 \\ 0 & 0 & m_1^2 \end{pmatrix}$$

Large off-diagonal elements are not good from FCNC.

Large breaking is not good.

symmetry breaking

Anyway, we have to break the symmetry to derive realistic lepton/quark masses and mixing angles.

When symmetry breaking is related with lepton masses,

e.g. $\Delta m_{12}^2 = m_\mu / m_1$

or more suppressed value,

that would be OK.

Such suppression could be obtained in string-inspired flavor models.

Ko, T.K., Park, Raby, ‘07

Ishimori, T.K., Ohki, Okada, Omura, Shimizu, Takahashi, Tanimoto, ’08, ‘09

Lepton flavor model building

First, we assume a certain flavor symmetry.

Then, we break it to a proper direction by flavon VEVs.

→ realistic MNS mixing matrix and lepton masses

We have achieved the first step for certain flavor symmetries.

Flavon VEVs correspond to deformation of compact space,
e.g. blow-up of orbifold singularity.

Which deformation is realistic ?

Another type of breaking, e.g.
by orbifold boundary conditions.

T.K., Omura, Yoshioka, '08

Summary

We have studied discrete symmetries and their anomalies.

We have just started non-Abelian discrete symmetries.

We have obtained limited discrete symmetries in heterotic orbifold models and intersecting/magnetized D-brane models.

What about string models on other compact spaces ?

Summary

It is still a challenging issues how to derive realistic quark/lepton mass matrices.

Flavon VEVs would correspond to a certain deformation from a symmetric compact space.